Some notes on precision track fit

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What I have done

- Implementing and testing a Kalman filter dealing with strong energy loss
- The test setups are simplified CMS and Belle 2 tracker geometries
- Everything was written in Matlab

Setup

One can specify these particle gun properties

- a particle mass
- a sign distribution
- Φ, z, θ distribution

and these detector properties

- strength of B_z
- L layers. Each has
- a position (radius only)
- measurement errors $\sigma_{R\Phi}$ σ_z
- a material (e.g.: Si)
- a thickness X

Testing

- To test the correctness of the track fit parameters $\boldsymbol{p}$ the standard score and a χ^2 test was used
- The tests were done separately for every layer and should produce:

$$\frac{p_{i,\mathrm{f}} - p_{i,\mathrm{t}}}{\sigma_{i,\mathrm{f}}} \sim \mathrm{Norm}(0, 1)$$

and

$$(\boldsymbol{p}_\mathrm{f} - \boldsymbol{p}_\mathrm{t})^\mathsf{T} \boldsymbol{C}^{-1} (\boldsymbol{p}_\mathrm{f} - \boldsymbol{p}_\mathrm{t}) \sim \chi^2_5$$

$$\boldsymbol{p}_\mathrm{e} - \boldsymbol{H}m(\epsilon)\boldsymbol{H}\boldsymbol{C}_\mathrm{e}(\boldsymbol{p}_\mathrm{e} - \boldsymbol{H}m)^\mathsf{T}$$

f: filtered
t: true

Material effects

- Energy loss was implemented using Bethe formula:

$$\Delta E(E) = \frac{n_e e^4}{4\pi m_e \epsilon_0^2} \left(\frac{1}{1 - \frac{m^2}{E^2}} \right) \left(\ln \left(\frac{2 m_e (E^2 - m^2)}{I m^2} \right) + \frac{m^2}{E^2} - 1 \right) \Delta x$$

- For testing purposes the energy loss was sometimes increased by a factor of 10
- And multiple scattering with Highland formula:

$$\sigma_\theta = \frac{0.015 E}{p^2} \sqrt{X} (1 + 0.038 \ln(X))$$

features needed to pass tests

- implicit calculation of ΔE in backward filter

$$E_{\text{new}} = \frac{E + \Delta E(E) - \frac{\mathrm{d}}{\mathrm{d}E}\Delta E(E) \cdot E}{1 - \frac{\mathrm{d}}{\mathrm{d}E}\Delta E(E)}$$

- energy loss not only increases κ but also σ_κ with linear error propagation and holding the correlations of $\boldsymbol{C}$ constant
- The E needed to calculate ΔE does not have to come from the acting filter but e.g. from the smoother or the reference track (necessary in the Belle 2 setup)

Parameter values for simulation

Belle 2 Setup:

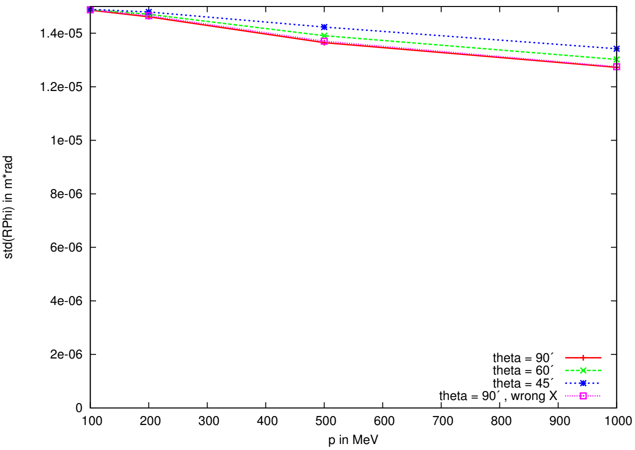
- $B_z = 1.5\,\mathrm{T}$
- radii: 1.3 2.2 3.8 8 11.5 14 (cm)
- $\sigma_{R\Phi}$: 15 15 10 15 15 15 (µm)
- σ_z: 20 20 30 50 50 50 (µm)
- thickness of Si layers: 1 1 1 1 1 1 (mm), 1 mm Si ≈ 0.0107 radiation lengths
- 20000 tracks each a single π^+
- Φ random between 2 and $2\pi - 2$, z random between -0.1 and 0.1
- p and θ same for all 20000 tracks

This results in $\Delta E(1\,\mathrm{GeV}) = 0.43\,\mathrm{MeV}$, $\Delta E(0.5\,\mathrm{GeV}) = 0.4\,\mathrm{MeV}$ $\Delta E(0.1\,\mathrm{GeV}) = 0.81\,\mathrm{MeV}$ per layer

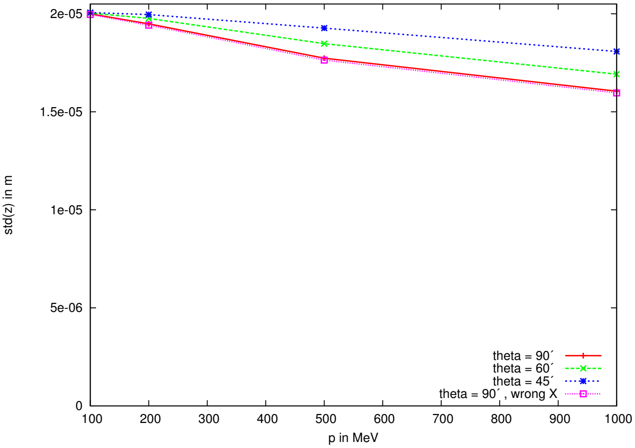
Some details of the filter

- the momentum used to calculate material effects comes from a reference track
- it is calculated like the simulation of a track but without the multiple scattering
- at the moment the reference track uses the track parameters of the simulation in layer 1
- this has to change, of course
- for some tests the filter uses wrong layer thicknesses (random errors of normal distribution with $\sigma = 0.1 \cdot X$)

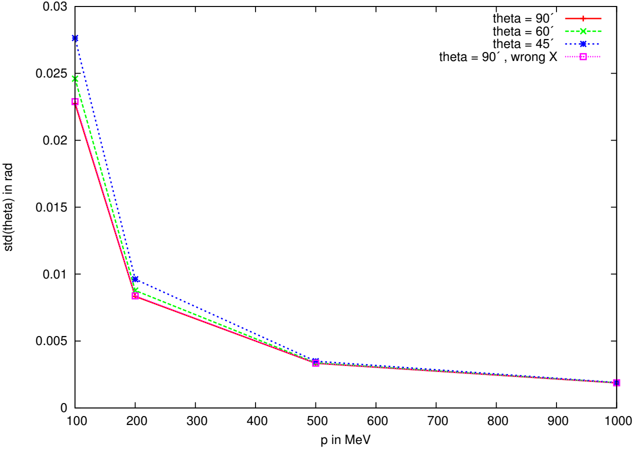
$\sigma_{R\Phi}$ in layer 1 at different initial particle momenta and different initial θ values:



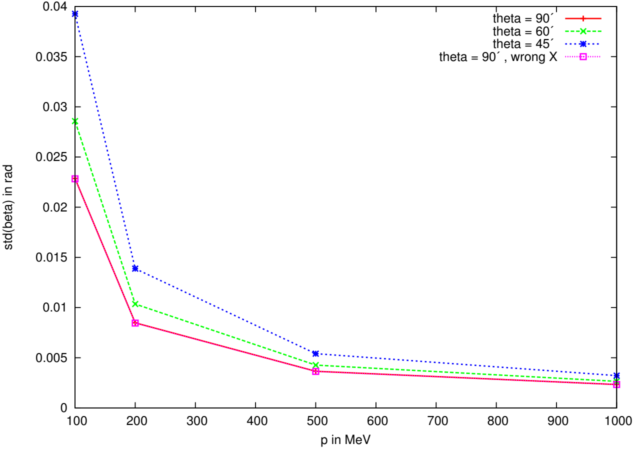
σ_z in layer 1 at different initial particle momenta and different initial θ values:



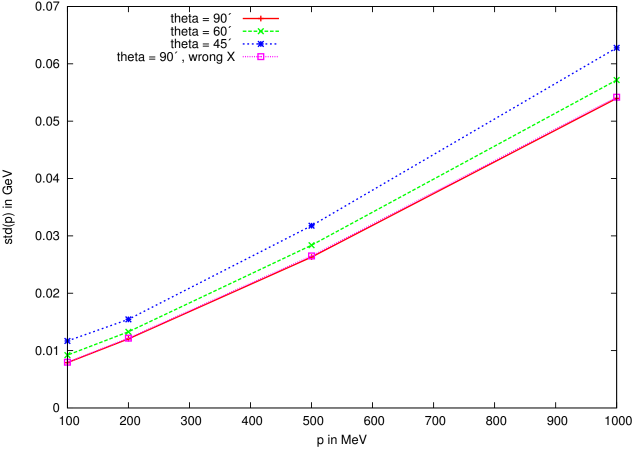
σ_θ in layer 1 at different initial particle momenta and different initial θ values:



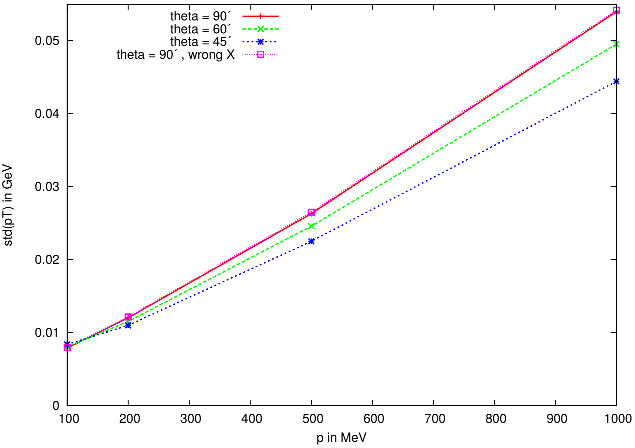
$\sigma_\beta = \sigma_{\phi - \Phi}$ in layer 1 at different initial particle momenta and different initial θ values:



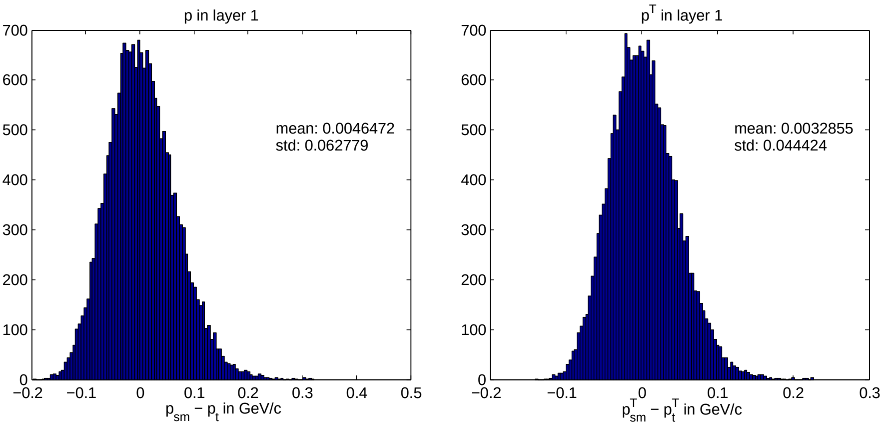
σ_p in layer 1 at different initial particle momenta and different initial θ values:



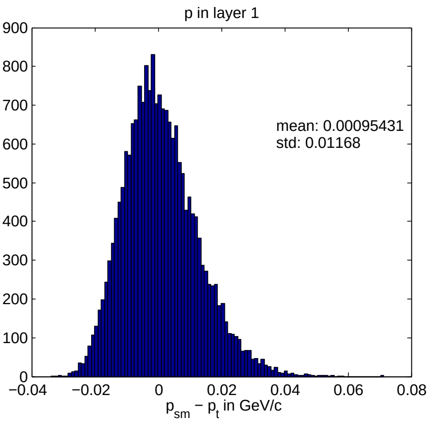
σ_{p_T} in layer 1 at different initial particle momenta and different initial θ values:

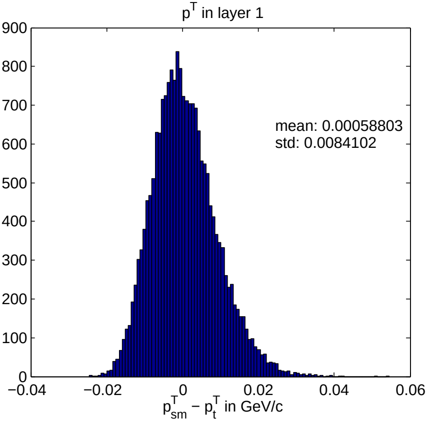


initial $p = 1$ GeV and $\theta = 45°$

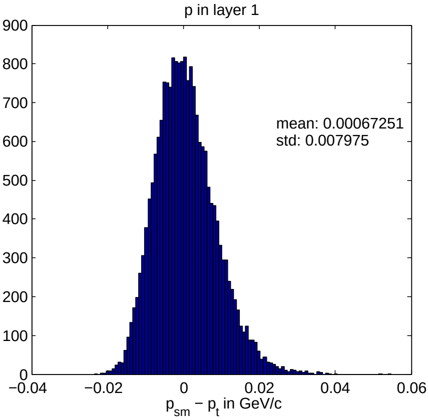


initial $p = 0.1$ GeV and $\theta = 45°$

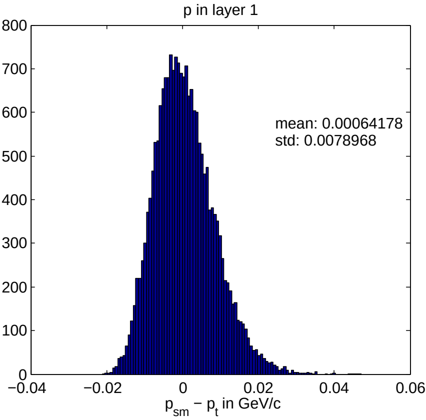




initial $p = 0.1$ GeV and $\theta = 90°$

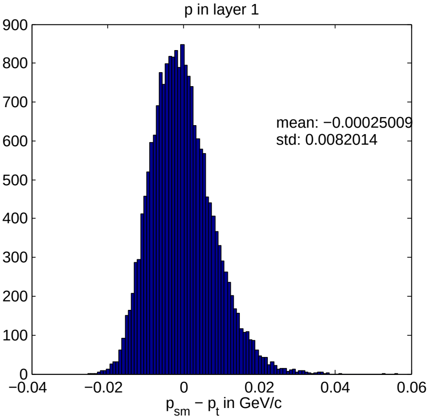


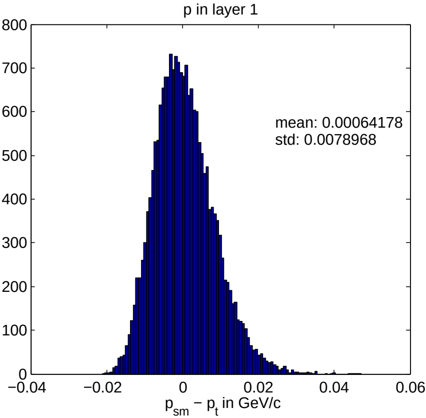
filter gets distorted X



filter gets correct X

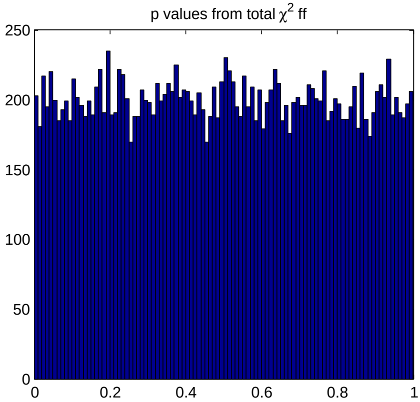
initial $p = 0.1$ GeV and $\theta = 90°$



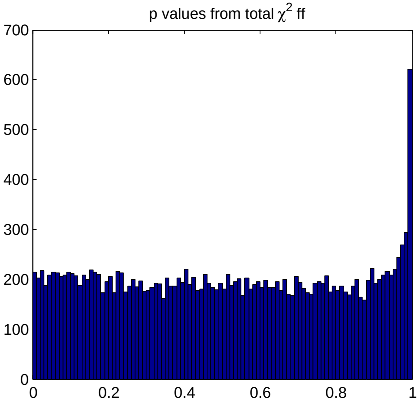


filter gets distorted X with 20% bias towards smaller values

filter gets correct X



initial $p = 1$ GeV and $\theta = 45°$



initial $p = 0.1$ GeV and $\theta = 45°$

Conclusions

- filter works over wide range of initial momenta and and angles
- filter produces correct results even with artifically increased energy loss
- only when multiple scattering gets too strong (small initial p and θ) a significant number of tracks a bad total chi^2 value
- random errors of 10% magnitude in X known to the filter have little influence on total χ^2 and momentum resolution in the inner layer

Outlook

- solve problem with strong multiple scattering
- investigate influence of wrong particle mass in filter
- change to a reference track that does not uses information from simulation

The End

This is the end